VICTORIA
COLLEGE
JERSEY

Physics Department

Lower Sixth Summer Project
For Paper 3A

Paper 3A is about interpreting and conducting practical work, especially graphs, this project sets out to address one of the three main strands of this.

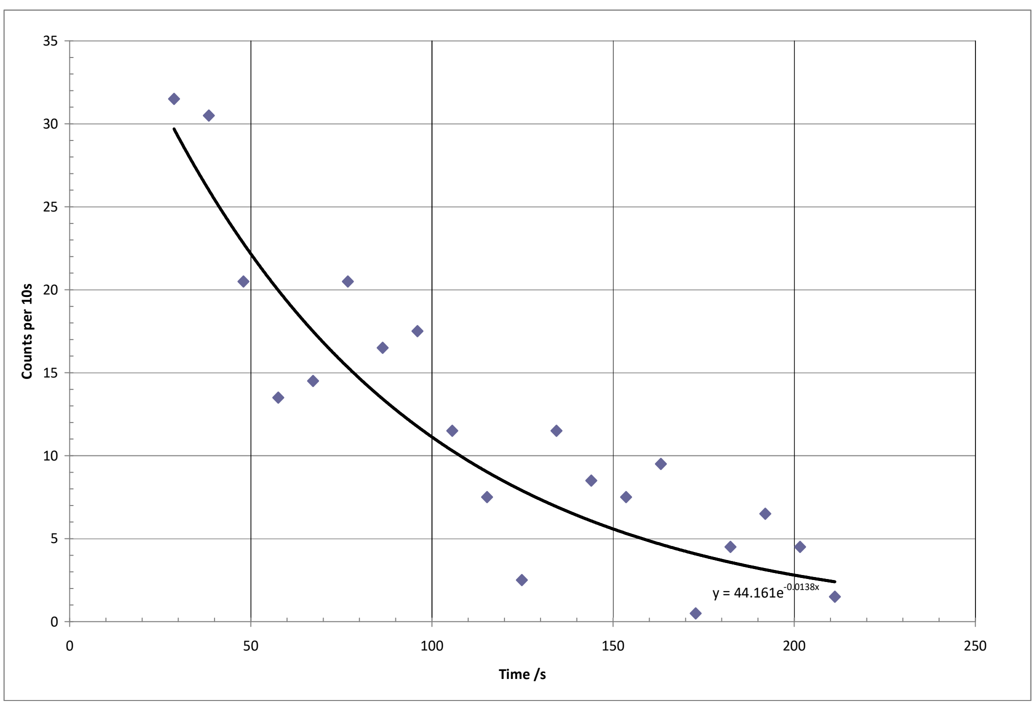
1. **Rearranging for and interpreting straight line graphs**
2. Determining Uncertainties
3. Designing and describing experiments

Rearranging for and Interpreting Straight Line Graphs

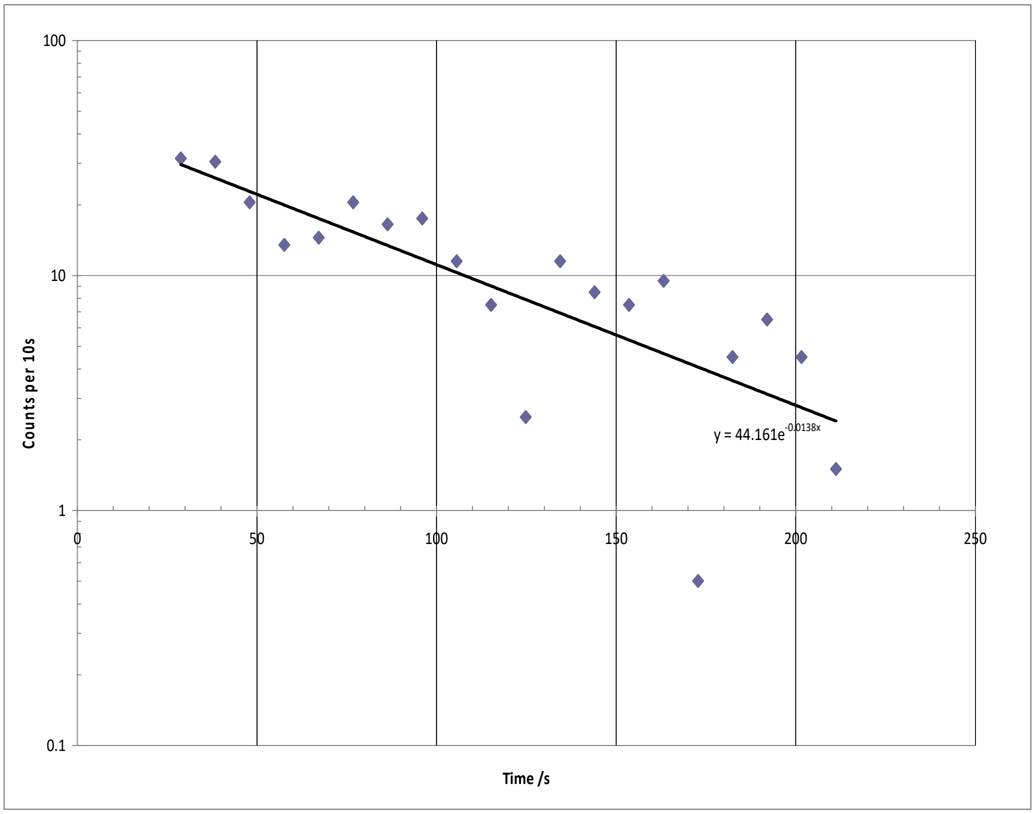
In the kinds of physics experiments performed at school, and very often in research, the end result is a graph from which it is hoped useful information can be extracted. Examples that you have met before include the resistance or resistivity of a wire from the gradient of its *V/I* or *R/L* lines or the spring constant from the gradient of the *F/Δx* graph.

More complex relationships will not generate a straight line in the first instance. It is perfectly possible for programs like Microsoft Excel to fit lines of best fit to curves and report the resulting parameters. However, it is hard, if not impossible, to judge by eye the quality of the fitting and the influence that unreliability in the data might be having on the fitting process (Excel cannot see and ignore anomalies the way that we would). For that reason there is still a benefit to be had from manipulating the data to produce a straight line where we can judge our own or the computer's fitting more easily, and thus extract parameters like a radioactive decay constant from the gradient or crossing point of the resulting graph. In the process you can build up a visual understanding of the reliability of the parameters.

For example, the graph below is from the measurement of the decay of protactinium (an experiment we do in Nuclear), the solid line is excel's fitting to a decay curve.



The curve looks fine, maybe a little low. But when the graph is manipulated to produce a straight line (by plotting the y-axis with a log scale), below, you can see immediately that a flatter line would be a better fit, especially if we assume that the bottom three points are anomalous.



At A2 three mathematical techniques to produce straight lines from curved data are required:

1. Power Law, where the power is known, use that knowledge to plot a straight line
2. Power law, where the power needs to be found, plot a log log graph.
3. Exponential Decay, where the law is exponential, take natural logs of the dependent variable.

The second two require knowledge of the behaviour of logarithmic functions which are now only taught in maths at A level, so those of you taking Physics without much maths will have to be careful to understand the introduction to logs later in this course and ask for help if you do not.

But for now: how to proceed where the power is known....

1 Where the Power is Already Known

Example 1, Known Power

Experimental variables x and y are believed to be related by the law $y = ax^2 + b$. How do we easily test that with a graph?

If we compare the likely relationship with the equation of a straight line

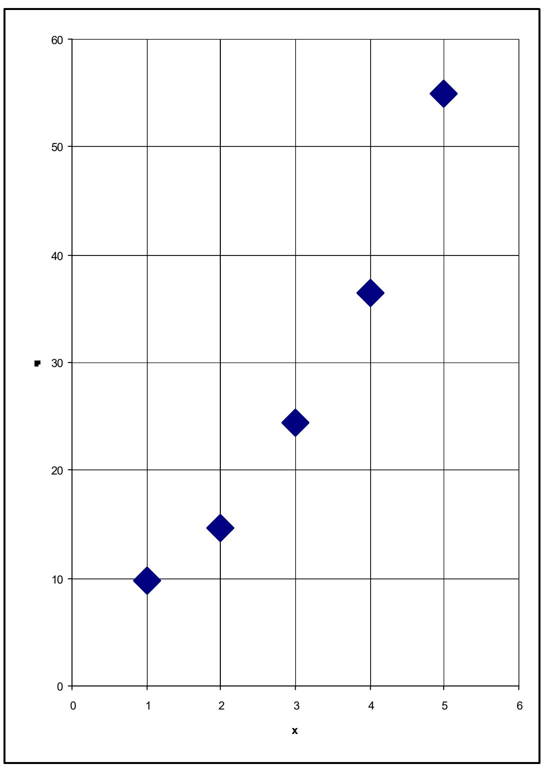
$$y = ax^2 + b$$

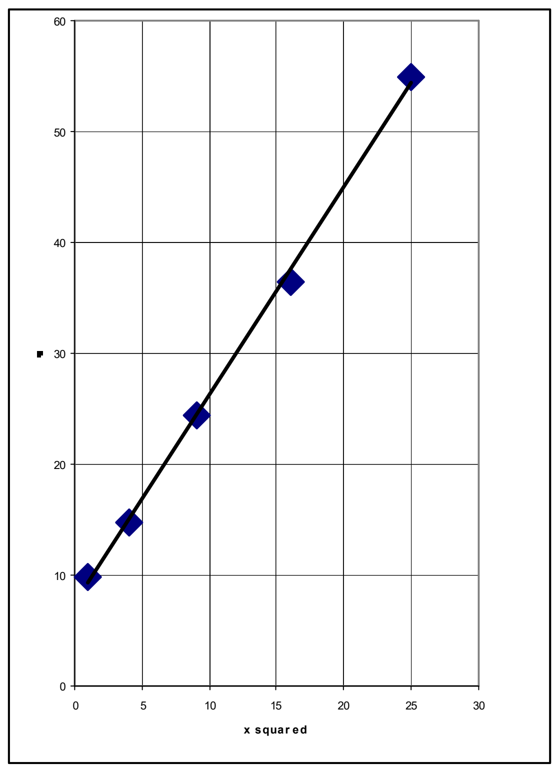
$$Y = mX + c$$

You can see that if we plot the variable y on the Y axis, but instead of x on the X, we plot x^2 on the X axis then the equations match. So:

x	1	2	3	4	5
x^2	1	4	9	16	25
y	9.8	14.7	24.4	36.5	55.0

Gives the graphs:





From which we can conclude that although there is a lit bit of scatter either side (the value for $x = 4$ might want rechecking) the fit of the straight line suggests that the $y=ax^2 + b$ model is appropriate.

If necessary we can then obtain the parameter a from the gradient of the x^2 graph and b from the crossing point.

Exercise 2, Known Power

For each of the likely rules connecting experimental variables x and y below state

a) What should be plotted on the vertical axis
b) What should be plotted on the horizontal axis
c) How the gradient matches the parameters in the relationship (i.e. for the first gradient = $-c$).
d) How the Y crossing point matches the parameters in the relationship

Relationship	a) Y axis	b) X axis	c) gradient	d) Y crossing
$y = d - cx^2$				
	a)	b)	c)	d)
$y = b\sqrt{x}$				
	a)	b)	c)	d)
$y - f = \frac{g}{x}$				
	a)	b)	c)	d)
$y - \frac{c}{x^2} = d$				
	a)	b)	c)	d)
$y = -gx^{-3}$				

Exercise 3 Known Power: Simple Harmonic Motion

A pendulum is an example of SHM the equation governing the time period T for a pendulum of length l is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Clearly in an experiment we would alter l and measure T so we would be looking to plot some version of l on the X axis and T on the Y axis. But, remembering that physicists do not like to deal in roots, we typically square the equation above:

$$T^2 = \frac{4\pi^2 l}{g}$$

In this modified version of the pendulum equation for a straight line what would we plot on the X axis?

……………….

What would we plot on the Y axis?

……………….

What would be the gradient?

……………….

Exercise 4 Known Power: some Engineering

The measured safe loads L in kN that may be applied to girders of varying spans, d in metres are found to be:

Span /m		Load /kN
1.60		588
2.00		475
2.80		339
3.60		264
4.20		226
4.80		198
5.40		175

It is thought that the relationship is inversely proportional i.e.

$$L \propto d^{-1}$$

Or including a constant of proportionality k,

$$L = kd^{-1}$$

How would you plot a straight-line graph to test this?

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Use the spare column in the table above to perform the necessary calculations for your test and plot your graph.

What is your constant of proportionality?

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What is the safe load for a 3m span?

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Would you be justified in working out the safe load for 6m from your graph? Why?

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Exercise 5, Known Power: a Simple Physics Example.

In an experiment the resistance of metre lengths of wire of the same material but different diameters were measured with the following results:

d /mm	0.52	0.76	1.10	1.42	1.75	2.04	2.56	3.08
R /Ω	5.95	3.12	1.64	1.14	0.89	0.76	0.63	0.55

R is related to *d* by the law:

$$R = \frac{a}{d^2} + b$$

The diameter is measured with a micrometer which has a resolution of one hundredth of a millimetre, so the smallest measurement of d can be expressed as:

$$0.52 \pm 0.01 \text{mm}$$

What is the percentage uncertainty in *d*?

………………

What is the percentage uncertainty in d^2?

………………

The corresponding value of *R* was calculated from the readings on an ammeter and a voltmeter

$$I = 0.20 \pm 0.01 \text{A}$$

$$V = 1.19 \pm 0.01 \text{V}$$

What is the percentage uncertainty in *R*?

………………

Choose what to plot on each axis and perform the calculations to appropriately fill in the blank line on the table, include the unit.

Use the completed table to draw a graph to confirm that the experimental values do match the given law.

Obtain values for parameters *a* and *b* from your graph, don't forget to express them in an appropriate number of significant figures and give units.

a =

………………

b =

………………

By comparing the relationship above with those for resistance and area:

$$R = \frac{\rho l}{A} \text{ and } A = \frac{\pi d^2}{4 \times 10^6}$$

Using the fact that l = 1m (the 10^6 term comes from d being measured in mm), it is possible to show that

$$a = \frac{4 \times 10^6 \times \rho}{\pi}$$

.

BUT what physical quantity does b represent in the experiment?

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Exercise 6 Known Power: an Experiment

The decrease in light intensity (I) as distance (r) from a point source increases follows what is called an inverse square law that is:

$$I \propto \frac{1}{r^2}$$

Or

$$I = I_1 r^{-2}$$

Where I_1 is a constant (the intensity at one metre).

The reason for this is straight forward; the intensity is effectively a measure of photons per unit area and the area that the photons are being spread out over goes up with the square of distance.

Design an experiment that uses a 12V lamp, a light meter attached to a datalogger and the normal equipment found in the lab to test this law.

Illustrate and describe the experiment below:

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Carry out a preliminary experiment to decide upon a range. *In this experiment where are the results going to be changing rapidly? Are even steps appropriate?* Having decided on your range and steps draw a table below. Do not forget to include space for your manipulation of one variable so that you can plot a straight line graph.

Have you considered how the ambient light will affect things?

Carry out the experiment and plot the graph

How well does your graph confirm the power law quoted above? Give an explanation of your answer

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What were the issues in measuring the variables in this experiment?

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Considering the point when you got your smallest average measurement of light intensity, what would you estimate the uncertainty in *r* was in your experiment?

……………….

Justify your answer

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Same questions for *I* (for which you should have repeats)?

……………….

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On the basis of your answers above calculate the percentage uncertainty in each

Percentage Uncertainty Light ………… Distance ……….

r is squared so its percentage uncertainty counts twice. In a classroom environment experiments with uncertainties below 5% can be considered reasonable experiments. How does this one match up?

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A note about Square Roots

In the second example of Exercise 2 you have probably said that $\sqrt{x}$ should be plotted on the x-axis. And that is perfectly correct, but traditionally physicists do not work with roots if they can avoid it so we would square the equation:

$$y^2 = b^2 x$$

In which case we would plot y^2 on the y-axis and x on the x-axis with b^2 being the gradient.

Today, with modern calculators, using the square root is a reasonable approach and so we have to decide which is best by thinking about likely problems or zero errors.

For example, in the experiment for Exercise 6 the measured light intensity will have a background value (a zero error) from the ambient light, we could subtract it, but assuming you didn't then the measured light I_m will be made up of two terms; the desired source intensity I_s and ambient I_a:

$$I_m = I_s + I_a$$

The overall equation is:

$$I = I_1 r^{-2}$$

And the traditional approach would be to plot I against r^{-2}, with I_1 as the gradient.

However, if we went with the square root option then:

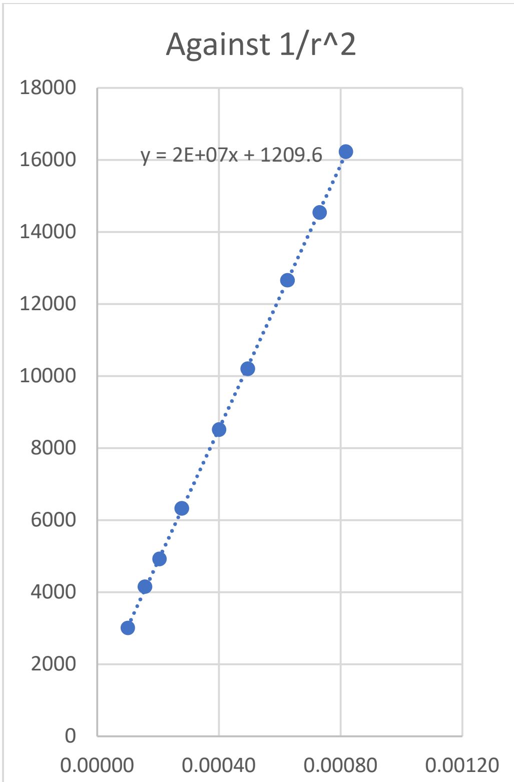
$$I^{-\frac{1}{2}} = I_1^{-\frac{1}{2}} \cdot r$$

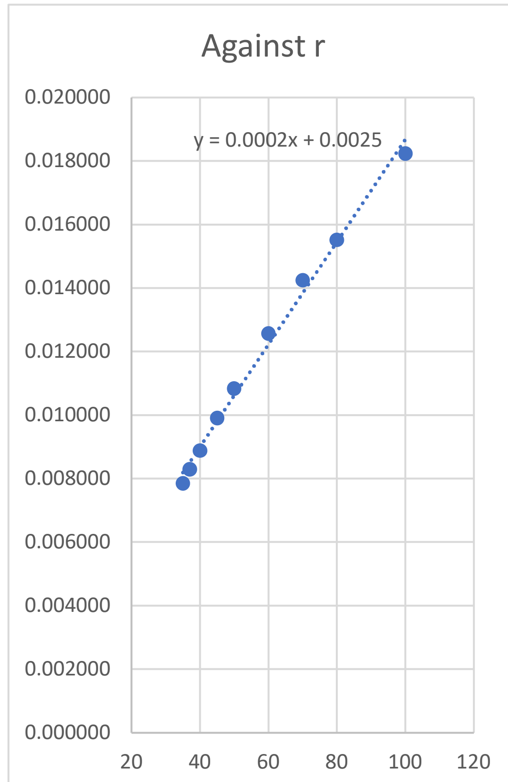
Where r would be on the x-axis and the gradient would be $I_1^{-\frac{1}{2}}$

If we try this out with some realistic values (I've exaggerated ambient a little):

r /mm	r^{-2} /mm^{-2}	I /lux	$I^{-\frac{1}{2}}$ /lux$^{-½}$
35	0.00082	16231	0.007849
37	0.00073	14540	0.008293
40	0.00063	12663	0.008887
45	0.00049	10200	0.009901
50	0.00040	8513	0.010838
60	0.00028	6327	0.012572
70	0.00020	4924	0.014251
80	0.00016	4153	0.015517
100	0.00010	3007	0.018236

The resulting graphs for the two approaches are on the next page. They should both give straight lines and yield the same values of I_1. However, you can see that the traditional approach gives the desired straight line, and the taking roots approach gives a curve. This is a result of the zero error (a constant) being part of the mathematically manipulated term (one over the square root).





Modify $I = I_1 r^{-2}$ using $I_m = I_s + I_a$ by working out which term is which, then identify the gradient term and the y-crossing term:

gradient =

y-crossing =

Attempt the same process for $I^{-\frac{1}{2}} = I_1^{-\frac{1}{2}} \cdot r$

In the Required Practical for Gamma Rays (RP12) this issue is the other way around. It is the value of r which has a large and hard to identify zero error and so here it is plotting $1/r^2$ on the x-axis which will result in a curve. For RP12 we have to take the alternative, "square root", approach.

Exercise 7 Known Power: an Experiment, an Extrapolation and a Required Practical (RP8.3)

For an ideal gas the relationship between temperature in ºC and volume in m^3 is:

$$V = kT + V_0$$

Where k and V_0 are constants.

An experimental set up for testing this is shown below:

Dry air is a reasonably ideal gas at normal temperatures. Set this experiment up, draw a table on the next sheet, and use the water bath to gently raise the temperature of the round bottomed flask.

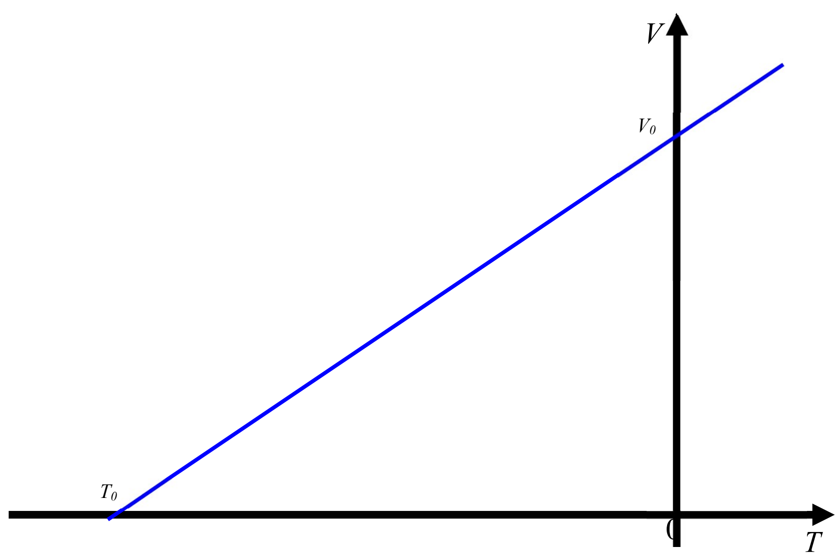
Measure the ***total air volume*** as the temperature changes.

You will need to think about how you are going to work out the total volume.

BE VERY CAREFUL WITH THE GAS SYRINGES, DO NOT PULL THEM APART AND STOP THE EXPERIMENT BEFORE THE PLUNGER IS PUSHED OUT OF THE END. The gas syringe plungers will need regular, gentle rotation so that they do not stick.

Draw your graph to test the relationship given above. As usual the plotted points must fill more than half the page.

If we were to do an extreme extrapolation of your results we could get a graph that looked something like:



If we alter the temperature scale such that zero degrees is at T_0 then T and V become directly proportional (Charles's Law). T_0 is therefore called absolute zero and that altered temperature scale is called the Kelvin scale. Use values from your graph to calculate your best value of T_0 in ºC

For such a big extrapolation a small uncertainty in gradient can have a big effect.

Uncertainties calculated by summing the percentage uncertainties of the total volume (what is the uncertainty in the measuring cylinder for example) and for temperature may well not capture the full extent of your uncertainty in the position of absolute zero.

On your graph draw in the line of best fit that most strongly departs from your best line, but which is reasonable with respect to your data. Use this line to calculate a new value for absolute zero.

A valid estimate of your uncertainty is the difference between these two values for absolute zero.

Write down your value for absolute zero including the uncertainty:

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How large is your uncertainty and is the true value (272.15°C) encompassed by your uncertainty?

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What do your answers say about your experiment?

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The questions on the previous page are representative of the type of thought about uncertainty which dominates the current A level spec. – however, there can also occasionally be combining multiple uncertainties questions, remember:

For variables being multiplied and divided, convert each uncertainty into a percentage and **add**.

For variables being added or subtracted, convert each uncertainty into an absolute or actual uncertainty and **add**.

The worst questions have you calculate uncertainties for each variable, some as percentage and some as absolute and then ask you to combine them. These are just to see who notices that some of the uncertainties are in the wrong format.

Here is a worked electricity example.

Example 8 Combining Uncertainties

A rectangular block of semiconductor is measured as having dimensions of 8mm, 6mm and 0.50mm. The long sides where measured with a 30cm ruler, the thickness was measured with a micrometer (measures to .01mm). The block's resistance was measured across the half millimetre thickness at 52Ω using an ohmmeter that is reported to be accurate within 5%.

What is the resistivity of the block, including the uncertainty?

Since the equation for resisitivity is $\rho = \frac{RA}{l}$ *all of the variables are being multiplied or divided. All the uncertainties therefore need to be in percentage form*

R = *52Ω ± 5%*

$h = .008m \pm (\frac{1}{8} \times 100)\%$ *(± one mm in eight)*

$w = .006m \pm (\frac{1}{6} \times 100)\%$ *(± one mm in six)*

$l = .0005m \pm (\frac{1}{50} \times 100)\%$ *(± one hundredth of a mm in fifty hundredths)*

The resistivity $= \frac{52 \times .008 \times .006}{.0005} = 5.0\Omega\text{m}$

The total % error is $\frac{1}{8} \times 100 + \frac{1}{6} \times 100 + \frac{1}{50} \times 100 + 5 = 36\%$

which just goes to show that something better than a ruler should have been used to measure the lengths, and that even two significant figures is unjustified in my answer.

The actual uncertainty is $5 \times 0.36 = \pm 1.8\Omega\text{m}$

So the final answer is:

5 ± 2 Ωm

2 Power law, where the power needs to be found

Logarithms

The technique for finding the power involves some maths that may not be familiar, especially if you are not doing maths to or past AS. The mathematical function to be used is called a logarithm.

It is not the function of this project to teach you logs, but as a quick introduction/reminder:

The logarithm function is properly written as $\log_a(b)$ where a is the base number of the logarithm and b is the number that the logarithm function is being performed on. The function returns the index (power) that a would have to raised to to give the number b.

$$\textbf{if} \quad b = a^c \quad \textbf{then} \quad \log_a b = c$$

For example:

$$2^6 = 64 \quad \textbf{so} \quad \log_2(64) = 6 \quad \textbf{and} \quad \text{antilog}_2(6) = 64$$

The thing that may surprise you is that logs usually return decimal answers. You are probably only used to thinking about indices or powers as whole numbers. Actually you have already met fractional indices because you will know that square root is the same as raising to the power half.

That is:

$$\sqrt{2} = 1.4142 \quad \textbf{and} \quad 2^{½} = 1.4142 \quad \textbf{therefore} \quad \log_2(1.4142) = 0.5$$

(*although you know that none of the above as I have written it is really exact because the square root of 2 is an irrational number*)

You can not take logs of zero or negative numbers. log of 1 will always be zero, because any number raised to the power zero is one, and log of any number between 0 and 1 will always be negative.

$$\frac{1}{\sqrt{2}} = 0.7071 \quad \textbf{and} \quad 2^{-½} = 0.7071 \quad \textbf{therefore} \quad \log_2(0.7071) = -0.5$$

I am sure that logarithms using different bases are very interesting to mathematicians, but actually a physicist or engineer only routinely uses two bases, base 10 and base e, we will come onto what e is later, from now on (although we could use any base) we will only use base 10 when trying to find the power involved in an experimental relationship.

Log to base 10 is so common that in mathematics there is a conventional shorthand:

$$\log_{10}(1000) = \lg(1000) = 3$$

AQA do not follow this convention and just use log without a base for $\log_{10}$

$$\log(0.001) = -3, \log(0.01) = -2, \log(0.1) = -1, \log(1) = 0, \log(10) = 1, \log(100) = 2 \text{ etc}$$

Your calculators do not follow the $\log_{10} = \lg$ convention either and like AQA call $\log_{10}$ just “log”. “Shift” “log” on your calculators will give you the antilog which is raising 10 to the power of the number you enter.

Exercise 9 Quick logs practice (How many sig figs should you use?)

Find the values of:

lg(27)	…………………	lg(0.27)	…………………
lg(7.32)	……………………	lg(732)	…………………
lg(0.005)	……………………	lg(0.5)	…………………
lg(1027.8)	……………………	lg(10.278)	…………………
lg(0.93)	……………………	lg(.0093)	…………………
lg(1.2×10^6)	……………………	lg(1.2)	…………………

Try antilogging your answers, use your rounded numbers to test the effect of the significant figures.

27	……………………	0.27	……………………
7.32	……………………	732	……………………
0.005	……………………	0.5	……………………
1027.8	……………………	10.278	……………………
0.93	……………………	0.0093	……………………
1.2×10^6	……………………	1.2	……………………

The short answer to how many sig figs you should use is that if you have to take logs in a table that AQA has given in an exam there will already be examples there. Copy the number of *decimal places* in the examples.

Significant figures follow across a table row, but decimal places trump that running down a table column.

The long answer is that there are many mathematical functions which quite strongly compress the number. Taking logs is an example, taking sine is another, if the number is further compressed by rounding, then when the opposite function is performed you may not recover anything like your original value. It makes sense therefore to go to one more significant figure for compressed values like logs than was in your original data, and AQA have tended to expect this in "complete the table" type questions in practical exams.

The mathematical rules for logs do not allow the number being logged to have a unit. In Physics we often want to log something with a unit and the convention is that we divide through by the unit first, so log of x ms^{-1} would look like:

$$\log(x \,/\, \mathrm{ms^{-1}})$$

You can tell the variable (the number) from the unit that is just there by convention because the variable is italic.

A frequently occurring, and confusing looking log in Physics is the log of V volts:

$$\log(V\,/\,\mathrm{V})$$

AQA always use this convention in questions, you just have to get used to it.

Just performing the log function on your calculator is not very interesting or useful, but there are three rules about the use of logs which are useful and which ***you have to know*** (these are true for any base).

1. Multiplied variables are added under logs

$$\log(a \times b) = \log a + \log b$$

2. Divided variables subtract under logs

$$\log\frac{a}{b} = \log a - \log b$$

3. Variables to a power are multiplied under logs (the power comes to the front)

$$\log a^b = b \times \log a$$

One consequence of these rules, which gets used a lot, is that reciprocals are equivalent to a negative under logs:

$$\log\frac{1}{a} = \log a^{-1} = -\log a$$

It is really easy to show that these rules are true using the rules for indices that you learnt in year 9, but you or your maths teacher can do that.

Example 10, Unknown Power, A Class Experiment and an RP

You should remember from GCSE that Boyle's Law says Pressure and Volume are inversely proportional.

$$P \propto \frac{1}{V}$$

Under logs this is:

$$\log P = \log V^{-1} + \log k$$

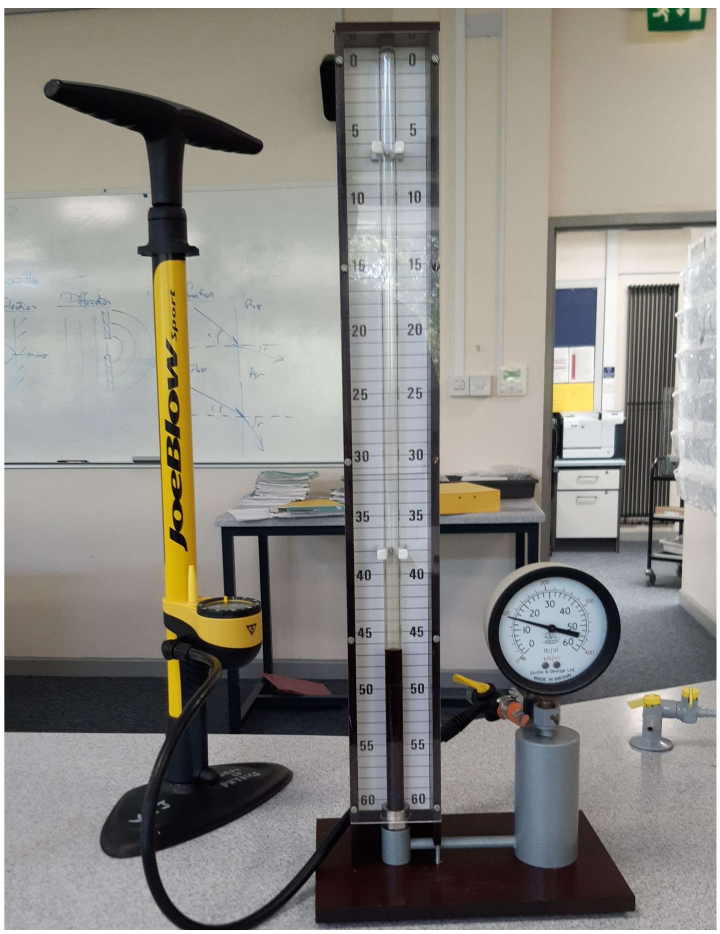
Where k is the constant of proportionality, allowing for the power this becomes:

$$\log P = -\log V \quad + \log k$$

Comparing this with $y = mx + c$ then we can demonstrate that Boyle's Law is true by plotting log$P$ against log$V$ and showing that the gradient is negative 1. The y-crossing, log$k$, is uninteresting in this case.

This process is using a log log graph (logs have been taken on both axes) to obtain the power (from the gradient) which relates two variables, here the power is minus 1, inversely proportional.

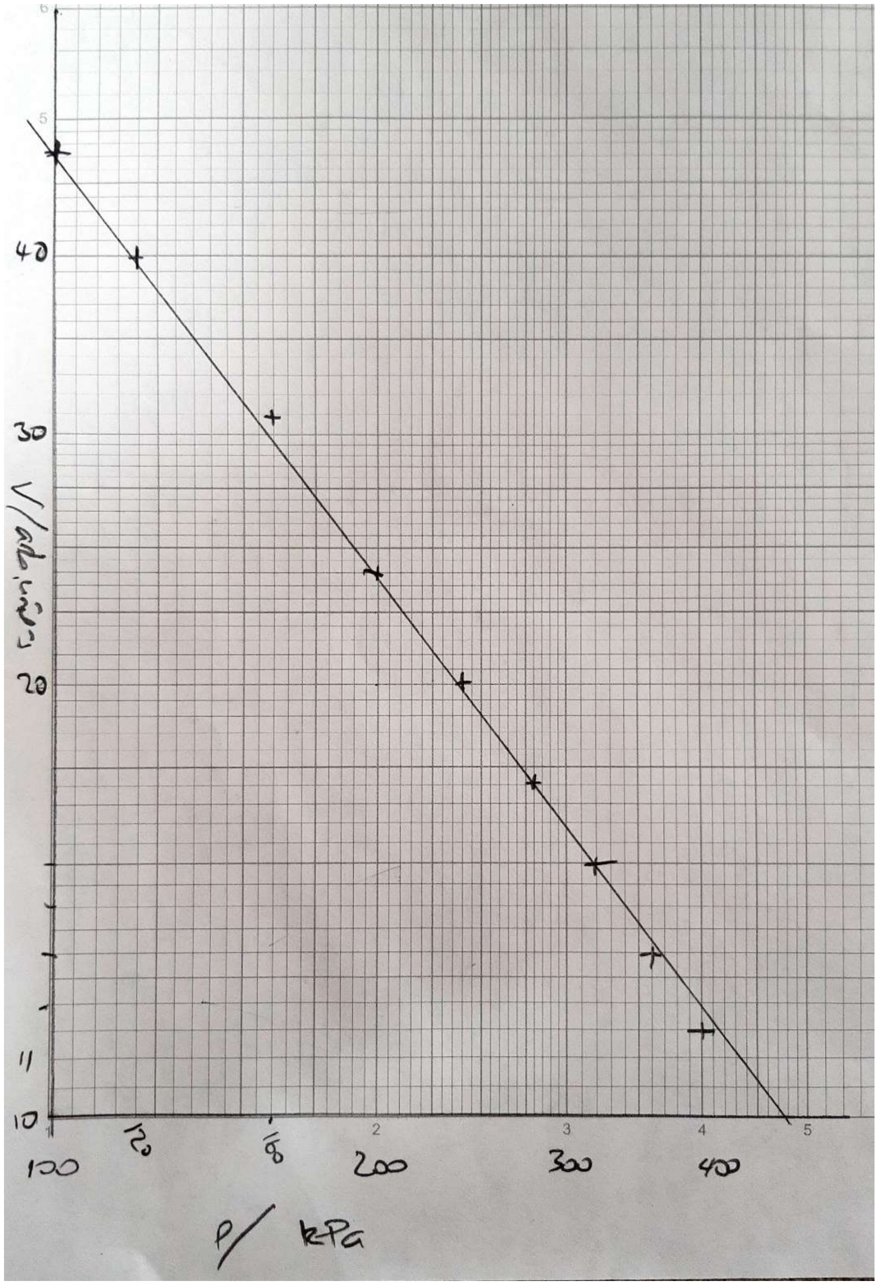
Confirming Boyle's Law is a class experiment and RP8.1



Often we treat RP8.1 as a known power experiment and plot V against P^{-1}

You can, however, do the above and show Boyle's Law is true by taking logs.

You could also plot the variables unaltered on log log paper.



This is special graph paper where the spacing between grid lines alters to match the compression that taking logs does to real numbers.

Log paper runs in decades, 1-10, where each represents an order of magnitude. So if the first 1-10 represents hundreds as in the x-axis above, then the next will be thousands – one thousand, two thousand, three..- the decade after that will be ten thousands and so on.

Exercise 11, Unknown Power, An Experiment

It is thought that the deflection from horizontal measured at the end of a cantilever (a beam supported at only one end) is a power law in terms of its unsupported length, because the deflection increases ever more rapidly as the length increases.

$$deflection \propto length^n$$

Using the metal strip provided, apply one hundred grams to act as the load, and obtain sufficient data to test whether it is indeed a power law and what that power is.

Once again you should consider whether even steps in your independent variable is the correct approach, whether you need repeats and how to avoid including zero errors into your results.

Carry out preliminary investigations and then design your table on a blank sheet to include all of your results as well as the columns for your graph; log(*length* /mm) and log (*deflection* /mm).

Does your graph support the power law hypothesis and if so what is the power?

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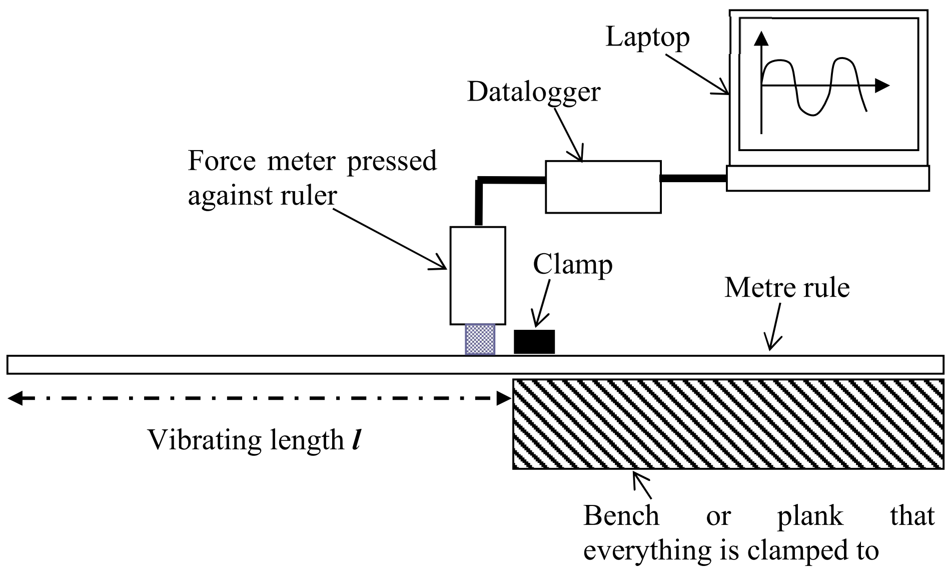
Cantilever Expt Table:

Exercise 12, Unknown Power, A More Complex Experiment

The relationship between the frequency of vibration of a ruler and the length of ruler that is vibrating is thought to be a power law.

Test this with the experimental set up below, with the laptops using the EasySense software on the Graph function. If you use the Interval button the trace on the laptop screen you will be able to determine the time period of each vibration.



[The force meter has a zeroing control that allows you to set the recorded force to zero at the centre position even though it is pressed against the ruler]

Note that the interval between recordings that you set on the laptop is important. It is affects your ability to discriminate the peaks in the recorded waveform and is the uncertainty in the time measurement.

Draw a table on the next page and record your results.

If you know your way around Excel, this one time, you can use Excel on the laptops (or your own) to take logs and even produce your graphs.

If you want to produce a graph on Excel you will need a pen drive because you can not print straight from the laptops and I still want hard copy. I also expect you to draw your own line of best fit, even if you get Excel to add a trend line as well.

Table for Change in Vibration Frequency with Length for a Metre Ruler

Does your graph support the power law hypothesis and if so what is the power?

………………………………………………………………………………………..
………………………………………………………………………………………..
………………………………………………………………………………………..
………………………………………………………………………………………..

That is it for Unknown Power. Find the power by logging the independent and dependent variables and finding the gradient of the resulting graph

3 Exponential Decay

The terms "exponential growth" or "exponential decay" have entered normal language, but in Maths and Science have specific meanings. They tend to be growth or decay curves where the rate of change *at that moment* depends upon the number of items there are *at that moment*. It turns out that curves of this type turn up frequently in nature; the rate of growth in the number of rabbits depends upon the number of rabbits, the rate of radioactive decay depends upon the number of radioactive isotopes present.

For exponential curves the independent variable, time in examples with rabbits or radiation, is found, not in the main equation, but in the power.

Any number can be used for the base, for example, we could describe nuclear decay as:

$$particles \propto 2^{-t/a}$$

Where here t is time and a, because we used 2 as the base, would be the half-life.

However, there is one base which has very useful mathematical properties when it comes to integration and differentiation: Euler's number. Exponential growth or decay is therefore almost always written as:

$$f(t) = A_0 e^{bt}$$

e is a universal constant (2.718..) which like others, such as pi, is irrational. A_0 is the value of the function at $t = 0$ and b is the growth, or if it is negative the decay, constant.

One simple way to tell an exponential decay from other similar looking curves like $1/x^2$ is that it has a half-life, which you learnt about at GCSE for radiation, but is true for all exponential curves.

As we have seen the mathematical opposite of applying a power to a known base is the logarithm, and so important is e as a base that, like 10, log to the base e has its own symbol.

$$\log_e(c) = \ln(c) = d \quad \Leftrightarrow \quad e^d = c$$

Logs to base e or ln are known as natural or Napierian logarithms. The "ln" button is next to the "log" button on your calculator, and e to the power …. is "shift" "ln".

Generally in Physics we know that a curve is exponential (the half-life test makes this very easy), what we are interested in is what the value of the decay constant is, and we obtain this by once again manipulating things to give a straight line.

$$A = A_0 e^{-bt}$$

Take natural logs

$$\ln(A) = \ln(A_0 e^{-bt})$$

Apply our log rules

$$\ln(A) = \ln(A_0) - bt\ln(e)$$

$\ln(e)$ must, by definition, equal 1

$$\ln(A) = \ln(A_0) - bt$$

A plot of $\ln(A)$ against t will have $-b$ as its gradient.

Exercise 13, Exponential Decay, Characterising a Thermistor

Thermistors are based upon semiconductors. Semiconductors have an energy gap (Eg) which the electrons must have the energy cross in order to be free to conduct. The proportion which have enough energy is given by the Boltzmann factor (k is the Boltzmann Constant):

$$\frac{n_m}{n} = e^{-E_g/kT}$$

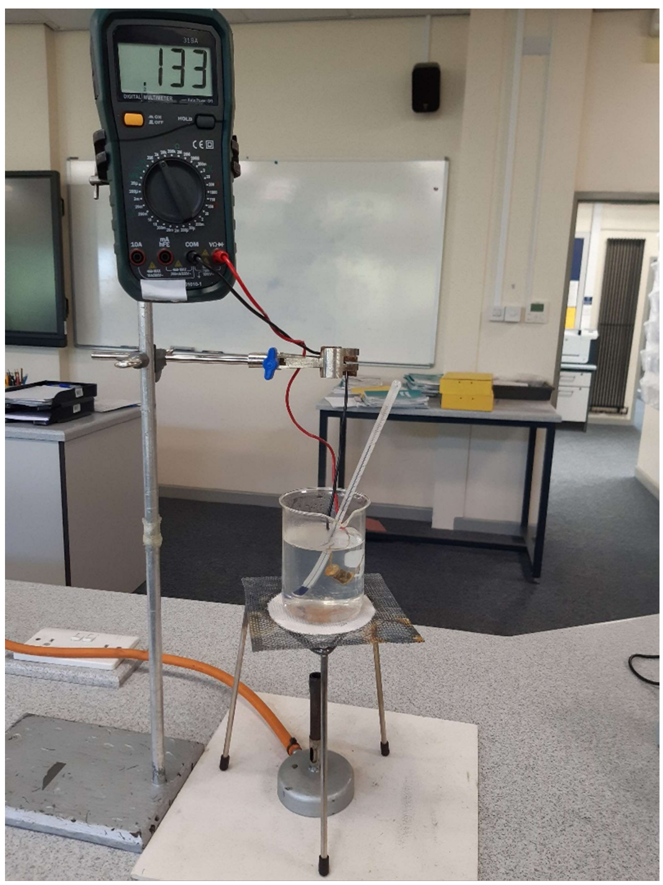
Resistance is inversely proportional to the number of free charge carriers therefore for a thermistor:

$$R \propto e^{E_g/kT}$$

Putting in a constant of proportionality (R_∞) and taking natural logs:

$$\ln R = \frac{E_g}{kT} + \ln R_\infty$$

Therefore plotting ln(R /Ω) against $1/T$ (T must be in Kelvin) for a thermistor, should produce a straight line graph whose gradient is given by E_g/k.



Set up the experiment as shown, using the multimeter as an ohmmeter, and record the thermistor's resistance from ice water to boiling, there is no need for repeats.

Table for Thermistor Expt (remember *T* in Kelvin)

Use your gradient and the value for Boltzmann's Constant in electron volts per Kelvin, 8.62×10^{-5} eVK^{-1}, to work out the energy gap for the semiconductor used in your thermistor :

Material	Energy gap (eV)	
	0K	300K
Si	1.17	1.11
Ge	0.74	0.66
InSb	0.23	0.17
InAs	0.43	0.36
InP	1.42	1.27
GaP	2.32	2.25
GaAs	1.52	1.43
GaSb	0.81	0.68
CdSe	1.84	1.74
ZnO	3.44	3.2
ZnS	3.91	3.6

Energy gap ……………………………

Looking at the table which material may well have been used in your Thermistor?

…………………………………………

Example 14, Exponential Decay, Modelling Radioactivity

In radioactivity the decay constant is the fraction of atoms likely to decay in a given unit time. [Decay constants have to have units which are the reciprocal of time because the index number itself has to be unitless.]

This can be modelled with what are called Tillich blocks. These are cubes with one side painted black, if a large number are thrown like dice then on average a sixth will have "decayed", i.e. will have the black side showing. We use this to model radioactivity by removing the "decayed" blocks after each throw, a throw is therefore standing for a unit of time. Either the number of blacks removed each time – the Activity of the radioactive sample – or the number remaining – the number of radioactive nuclei - can be recorded. Both decay exponentially. Because the decay of one nucleus or one cube is random a smooth exponential decay relies on having lots of cubes or nuclei. For nuclei that's fine, one gram of U-238 contains 2.5×10^{21} nuclei, but for blocks…

Take at least 150 blocks. Throw them removing the dark side up ones each time and record the numbers. Plot a graph of ln(number) against number of throws. What is your gradient?

Gradient = ………………

For exponential graphs the gradient of the natural log graph is the rate of change of the population.

For a thermistor this is the change in the population of free electrons per degree and consequently the rate in change of resistance. For exponential decay over time, as in radioactivity, the gradient represents the proportion of the population which should on average decay in each time period. For the case of dice with one side coloured in the proportion which should be removed (should decay) in every throw is a sixth. Therefore, the decay constant, the negative of the gradient, (if you did not include the negative sign in your gradient you have lost marks) should be 0.167 per throw.

For Exponential Relationships, plot natural log of the dependent against the independent. The negative of the gradient is the decay constant.

In Summary:

Using straight-line graphs to obtain data is fundamental to how Experimental Physics is performed.

There are three common approaches:

- Where you know the formula, or you think you know the formula and want it confirming, mathematically modify the experimental variables with y equals mx plus c in mind so that when they are plotted a straight-line graph results.
- Where you think the relationship between two variables is proportional, but with one of the variables raised to a power, plot a log-log graph and the gradient should be the power.
- Where the relationship between two variables is exponential, plot natural log on the y-axis and the resulting gradient should be the rate of change to the population per whatever is on your x-axis.

Bear in mind that a curve does not necessarily negate the applicability of the formula; it may be that systematic error is being included into your mathematical modification of the variables and is producing the curve. If you have an idea what the systematic error might be then it may be that a different mathematical approach will enable you to mitigate its effects.

Finally, uncertainties are complex, but our exam board has taken two approaches:

- Combine individual uncertainties for that row of results which is likely to have the largest percentage uncertainty. The result is an estimate of the uncertainty for the experiment (most common in the previous spec.).
- Use the extremes of the possible lines of best fit which are compatible with the plotted data to define an uncertainty range for a gradient or for an intercept (most common in the current spec. - particularly compatible with the use of error bars on the plots).